Supersymmetric AdS Solutions in Supergravity

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Introduction

- Supersymmetric solutions in supergravity are well studied, many are classified.

- Anti-de Sitter solutions appear very frequently, attracted much interest since the advent of AdS/CFT.

This talk

- AdS_7 backgrounds in gauged supergravity and their moduli space.
- General criteria for the existence of **AdS** solutions that do **not break** any **supersymmetry**.

Supersymmetric backgrounds in supergravity

- SUSY variation of the gravitini:

$$\delta\psi^i_M = \mathrm{D}_M\epsilon^i + \sum_p \left(\alpha_{(p)}\right)^i_{\ j} F^{(p)}_{N_1\dots N_p} T^{N_1\dots N_p}{}_M\epsilon^j + A^i_{0\,j}\Gamma_M\epsilon^j$$

- SUSY variation of spin 1/2 fermions:

$$\delta\chi^a = \sum_p \left(\beta_{(p)}\right)^a_{\ i}\ F^{(p)}_{N_1\dots N_p}\Gamma^{N_1\dots N_p}\epsilon^i + A^a_{1\,i}\,\epsilon^i$$

$\rightarrow$ Potential:

$$V = -\#\mathrm{tr}(A_0^2) + \#\mathrm{tr}(A_1^2)$$

Supersymmetric backgrounds:

$$\langle\delta\psi^i_\mu\rangle = \langle\delta\chi^a\rangle = 0$$

Supersymmetric backgrounds without fluxes

- Supersymmetric solutions of gauged supergravity without background fluxes need to satisfy the Killing spinor equation:

$$\delta\psi^i_M = \nabla_M \epsilon^i + A^i_{0\,j} \Gamma_M \epsilon^j = 0$$

- Integrability condition:

$$\left(\frac{1}{4} R_{MN}{}^{AB} \delta^i_k + 2 A^i_{0\,j} A^j_{0\,k} \delta^A_M \delta^B_N \right) \Gamma_{AB} \epsilon^k = 0$$

$\Rightarrow$ Unbroken supersymmetry (without fluxes):
$\mathbf{Mink_d} \times \mathbf{T^{(D-d)}}$ and $\mathbf{AdS_D}$ are the only possible solutions.

- For $D \leq 7$ *most* gauged supergravities admit AdS_D solutions.
[de Alwis, Louis, McAllister, Triendl, Westphal '13] [Louis, Triendl '14]
[Louis, SL '15] [Louis, Triendl, Zagermann '15] [Louis, Muranaka '16]

Example: AdS_7 solutions in half-maximal SUGRA in $D = 7$

[Louis, SL '15]

Field content of $\mathcal{N} = 2$ supergravity

- gravity multiplet

$$\left(e_\mu^m, \psi^A, A_\mu^i, \chi^A, B_{\mu\nu}, \sigma\right) , \qquad A = 1, 2\,, \quad i = 1, 2, 3$$

- n vector multiplets

$$\left(A_\mu^r, \lambda^{rA}, \phi^{ri}\right) , \qquad r = 1, \ldots, n$$

- AdS_7 solutions require a non-trivial potential

⇒ **Gauged supergravity**

Gauged half-maximal supergravity in $D = 7$

- vectors: $A^I = (A^i, A^r)$, $I = 1, \dots, 3 + n$
- scalars: $\{\sigma, \phi^{ir}\}$, span the scalar field space:

$$\mathbb{R}^+ \times \frac{SO(3,n)}{SO(3) \times SO(n)}$$

$\Rightarrow$ Global symmetry group: $\mathbf{R}^+ \times \mathbf{SO(3,n)}$

- **gauge** a subgroup:

$$G \subset \mathbb{R}^+ \times SO(3,n)$$

$\Rightarrow$ generation of non-trivial potential $V(\sigma, \phi^{ir})$

Supersymmetric AdS_7 solutions

- A supersymmetric solution needs to satisfy

$$\langle \delta\psi_\mu \rangle = \langle \delta\chi \rangle = \langle \delta\lambda^r \rangle = 0$$

$$\Rightarrow \quad A_0 \sim \sqrt{\langle V \rangle}\,, \quad A_1 = 0\,.$$

$\Rightarrow$ For $\langle V \rangle < 0$: Conditions on the structure constants of G:

$$f_{ijk} = g\epsilon_{ijk}\,, \quad f_{ijr} = 0$$

Solution: possible gauge groups

$$G = G_0 \times H = \left\{ \begin{array}{l} SO(3) \\ SO(3,1) \\ SL(3,\mathbb{R}) \end{array} \right\} \times H\,,$$

with $H \subset SO(n)$ arbitrary.

(See also: [Karndumri '14])

The Moduli Space

- Find flat directions $\delta\phi_{ir}$ that preserve SUSY conditions.

- Result:

$$\delta\phi_{ir} = f_{irs}\lambda^s \,,$$

  for λ^s arbitrary.

- f_{irs} correspond to non-compact directions of gauge group G

$$\Rightarrow \quad \mathcal{M}_{\delta\phi} = \frac{G_0}{SO(3)}$$

Spontaneous breaking of the gauge group

- Evaluated in the vacuum the Lagrangian contains a term of the form

$$\mathcal{L} \supset \left(D\phi^{ir}\right)^2 \supset (f_{irs}A^s)^2$$

Spontaneous breaking of the gauge group

$$G = \left\{ \begin{matrix} SO(3) \\ SO(3,1) \\ SL(3,\mathbb{R}) \end{matrix} \right\} \times H \quad \rightarrow \quad \mathbf{SO(3)} \times \mathbf{H}\,.$$

- All massless scalars $\delta\phi_{ir}$ are Goldstone bosons and are "eaten" by massive vectors.

True moduli space

$$\mathcal{M}_{AdS} = \left\{ \begin{matrix} \text{isolated} \\ \text{points} \end{matrix} \right\}$$

Comparison with the dual SCFT

AdS/CFT correspondence

$\mathcal{N} = 2$ SUGRA on AdS_7 $\Leftrightarrow$ $d = 6$, $\mathcal{N} = (1,0)$ SCFT on the boundary of AdS_7

- gauge symmetry: $SO(3) \times H$ $\Leftrightarrow$ global symmetry: $SU(2)_R \times H_{\text{flavour}}$
- scalar field with mass m $\Leftrightarrow$ conf. operator of dimension

$$\Delta = \frac{6}{2} + \sqrt{\left(\frac{6}{2}\right)^2 + m^2 L^2}$$

Conformal manifold

- Deform a SCFT by operators $\mathcal{O}_i$:

$$S \to S + \sum_i \int \varphi^i \mathcal{O}_i$$

- Deformations that do not break superconformal invariance: exactly marginal operators

- Conformal manifold $\mathcal{C}$ = space of exactly marginal couplings φ^i.

Conjecture from AdS/CFT:

$$\mathcal{M}_{AdS} \cong \mathcal{C}$$

- Indeed: There are **no marginal operators** in six-dimensional SCFTs [Louis, SL '15]

Are there other maximally supersymmetric backgrounds without fluxes?

$\rightarrow$ **Back to the general analysis.**

- What about backgrounds of **product form**?

$$\mathcal{M}_D = \mathcal{M}_d \times Y^{(D-d)}$$

- For unbroken supersymmetry:

$$R_{MNAB} = -\frac{4}{\mathcal{N}} \mathrm{tr}\left(A_0^2\right) \left(g_{MA} g_{NB} - g_{MB} g_{NA}\right) .$$

- This is only of product form if $\mathrm{tr}\left(A_0^2\right) = 0$.

$\Rightarrow$ $\mathcal{M}_d$ cannot be **AdS** (For $d \neq D$)

Flux compactifications

- Try to construct backgrounds of the form

$$\mathcal{M}_D = AdS_d \times Y^{(D-d)}$$

$\Rightarrow$ Introduction of **background fluxes**:

$$\delta\psi_M = \mathrm{D}_M\epsilon + \sum_p \alpha_{(p)} F^{(p)}_{N_1\dots N_p} T_{(p)}^{N_1\dots N_p}{}_M\epsilon + A_0\epsilon$$

- **Unbroken supersymmetry** requires top-form fluxes:

$$F^{(d)} \sim \mathrm{vol}_d \qquad \text{or} \qquad F^{(D-d)} \sim \mathrm{vol}_{(D-d)}$$

and $A_0 = 0$.

$\Rightarrow$ Freund-Rubin compactification: $\mathcal{M}_D = \mathbf{AdS_d} \times \mathbf{S^{(D-d)}}$

Additional constraints by supersymmetry

- Supersymmetry variations of additional spin-1/2 fermions in the gravity multiplet:

$$\delta\chi = \sum_p \beta_{(p)} F^{(p)}_{M_1\dots M_p} \Gamma^{M_1\dots M_p}\epsilon + A_1\epsilon$$

$\Rightarrow$ Supersymmetry gets (partially) broken.

- Exception: (anti-)self-dual fluxes in chiral theories.

- Notice: Fermions in other multiplets than the gravity multiplet do not cause any problems.

Conditions for supersymmetric AdS solutions

Fully supersymmetric solutions of supergravity containing an AdS factor are always of the form

$$AdS_d \times S^{(D-d)}\,.$$

Such solutions are only possible if the gravity multiplet contains a form field of rank $d-1$ or $D-d-1$. Moreover, unless the corresponding field strength satisfies a (anti-)self-duality condition in a chiral theory, the gravity multiplet must not contain any spin-1/2 fermions.

Survey of AdS solutions with unbroken supersymmetry

Complete classification of $\mathbf{AdS_d \times S^{D-d}}$ solutions:

$D = 11$	$\mathcal{N} = 1$	$AdS_4 \times S^7$, $AdS_7 \times S^4$
$D = 10$	IIB	$AdS_5 \times S^5$
$D = 6$	$\mathcal{N} = (1,0)$ $\mathcal{N} = (2,0)$	$AdS_3 \times S^3$
$D = 5$	$\mathcal{N} = 2$	$AdS_2 \times S^3$, $AdS_3 \times S^2$
$D = 4$	$\mathcal{N} = 2$	$AdS_2 \times S^2$

Conclusions

- The only non-supersymmetry breaking AdS solutions in supergravity are of the form

$$\mathbf{AdS_D} \qquad \text{or} \qquad \mathbf{AdS_d \times S^{D-d}}$$

- AdS_D appears generically in gauged supergravities.
- Example: half-maximal gauged supergravity in $D = 7$.

- It can be seen directly from the field content of the gravity multiplet whether solutions of the form $AdS_d \times S^{D-d}$ exist.