New Modes from Higher Curvature Corrections in Holography

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References

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Higher derivative corrections

- Arise in string theory and play a role in quantum gravity
- Improve renormalizability
- Lead to ghosts
- May lead to violations of causality
- In AdS/CFT context are believed to be dual to non-unitatry or acausal field theories.

Motivation

- Can one have consistent theories of gravity involving higher curvature corrections or massive gravitons? How to achieve consistency?
- What does holographic duality tells us about the higher derivative corrections to GR?

Our Approach

We use the holographic dictionary to obtain basic properties of the new modes/dual operators. In particular we study how new modes modify the near-boundary expansion and effect the physics of the stress-energy tensor.

Outline

- Setup and some basic features
- Physics of the new modes
- Near-boundary analysis
    - Universality of GR/Scale vs. Conformal invariance
    - Gravitational Chern-Simons theory in 5d

Variational problem

As clarified in [Skenderis, Taylor, van Rees (2009)] and emphasized in [Smolic, Taylor (2013)] the higher derivative terms in the action generically lead to the new degrees of freedom. These have to be taken care of when setting up the variational problem. Imposing $\delta g = 0$ at the boundary of AdS is not enough to set the variational problem. Other sources have to be fixed as well.

The Model

$$S = \frac{1}{16\pi G_{d+1}} \int d^{d+1}x\sqrt{-G}\Big[R + \frac{d(d-1)}{L^2} + L^2(\lambda_1 R_{abcd}R^{abcd} + \lambda_2 R_{ab}R^{ab} + \lambda_3 R^2)\Big].$$

Our goal here is to perform the Fefferman-Graham / Henningson-Skenderis type analysis of this theory

Presence of an AdS vacuum

Looking for a solution of the form

$$ds^2 = dr^2 + e^{2r/l}\eta_{ij},$$

we find a fourth order characteristic equation for the radius of AdS l. The solution is

$$\frac{4}{l^2} = \frac{1 \pm \sqrt{1 - 64\lambda}}{8\lambda L^2}$$

with

$$\lambda = \frac{d-3}{8(d-1)}\Big(\lambda_1 + \frac{d}{2}\lambda_2 + \frac{d(d+1)}{2}\lambda_3\Big).$$

The smaller root is continuously connected to the pure AdS solution of Einstein's gravity.

New modes

Field equations are fourth order in derivatives, hence we expect that there are four independent boundary conditions: two sources and two VEVs.

We parametrize the metric as

$$ds^2 = dr^2 + \gamma_{ij}(x,r)dx^i dx^j,$$
$$\gamma_{ij}(x,r) = e^{2r/l}(g_{(0)ij}(x,r) + e^{-nr/l} g_{(n)ij}(x,r) + \dots)$$

New modes

The trace equation gives

$$n(d+1-n)a_n \mathrm{tr}(g_{(n)}) = 0,$$

$$a_n = 1 + 4\frac{L^2}{l^2}\Big(-8\lambda + \frac{n}{2}\frac{d-n}{d-1}(2\lambda_1 + \frac{d+1}{2}\lambda_2 + 2d\lambda_3)\Big),$$

The (ij)-equation gives

$$n(n-d)\hat{a}_n g_{(n)ij} + \ldots = 0,$$

$$\hat{a}_n = 1 - \frac{L^2}{l^2}\Big(32\frac{d-1}{d-3}\lambda + n(d-n)(4\lambda_1 + \lambda_2) + 4(2-d)\lambda_1\Big).$$

New modes

Values of n for which a_n or $\hat{a}_n$ vanish indicate (dimensions of) new modes

Fefferman-Graham Expansion

We consider theory on a general background $g_{(0)ij}(x)$

We parametrize the metric as

$$ds^2 = dr^2 + e^{2r/l} g_{ij}(x,r)\, dx^i dx^j,$$
$$g_{ij} = g_{(0)ij} + e^{-2r/l} g_{(2)ij} + e^{-3r/l} g_{(3)ij} + e^{-4r/l} g_{(4)ij} + \dots.$$

and solve field equations order by order near the boundary

n=2

At the next to leading order (of direct interest when $d = 2$) we find

$$a_2(\lambda_i)(l^2 R_{(0)} + 2(d-1)\mathrm{tr}(g_{(2)})) = 0,$$

$$\hat{a}_2(\lambda_i)\Big(\nabla_i \mathrm{tr}(g_{(2)}) - \nabla^j g_{(2)ij}\Big) + \ldots = 0,$$

$$\hat{a}_2(\lambda_i)\Big[g_{(2)ij} - \frac{l^2}{d-2}\Big(\frac{R_{(0)} g_{(0)ij}}{2(d-1)} - R_{(0)ij}\Big)\Big] + \ldots = 0,$$

n=2: different cases

- **In the case** $a_2(\lambda_i) \neq 0$ **and** $\hat{a}_2(\lambda_i) \neq 0$ the solution to the equations above coincides with the well known result for GR:

$$\mathrm{tr}(g_{(2)}) = -\frac{l^2 R_{(0)}}{2(d-1)},$$
$$\nabla^j g_{(2)ij} = \nabla_i \mathrm{tr}(g_{(2)}),$$
$$g_{(2)ij} = \frac{l^2}{d-2}\Big(\frac{R_{(0)}}{2(d-1)} g_{(0)ij} - R_{(0)ij}\Big)$$

- **In the case** $a_2(\lambda_i) \neq 0$ **and** $\hat{a}_2(\lambda_i) = 0$ the trace $\mathrm{tr}(g_{(2)})$ is still as in GR however $\nabla^j g_{(2)ij}$ and $g_{(2)ij}$ are left undetermined.

n=2: different cases

- **In the case** $a_2(\lambda_i) = 0$ **and** $\hat{a}_2(\lambda_i) \neq 0$ the trace $\mathrm{tr}(g_{(2)})$ is left undetermined, whereas $\nabla^j g_{(2)ij}$ and $g_{(2)ij}$ are determined. **Example:** new massive gravity in 3d at the critical point [E.A. Bergshoeff, O. Hohm, P.K.Townsend (2009)] is a particular member of this special family.
- **Finally in the case** $a_2(\lambda_i) = 0$ **and** $\hat{a}_2(\lambda_i) = 0$ nothing gets determined. This case cannot be realised if $d = 2$ or $d = 3$ if the R-term is present in the action. However the conformal gravity in $d = 4$ is an example of this scenario. In this case $a_n = \hat{a}_n = 0$ for $n = 1$ and $n = 2$. This confirms the proposed ansatz [D. Grumiller et al. (2014)] for the near-boundary expansion.

At the next order (of direct interest when $d = 3$) we find

$$a_3(\lambda_i)\mathrm{tr}(g_{(3)}) = 0,$$
$$\hat{a}_3(\lambda_i)\Big(\nabla_i \mathrm{tr}(g_{(3)}) - \nabla^j g_{(3)ij}\Big) + \ldots = 0,$$
$$\hat{a}_3(\lambda_i)\Big[(d-3)g_{(3)ij} + \mathrm{tr}(g_{(3)})g_{(0)ij}\Big] + \ldots = 0,$$

$a_3 = 1$ in $d = 3$.

Example with $\hat{a}_3 = 0$: logarithmic point of the critical gravity in four bulk dimensions [H. Lu, C.N. Pope (2011)]. Graviton acquires logarithmic partner at this point.

n=4

The trace equation for $g_{(4)ij}$ (of direct interest when $d = 4$) is

$$a_4(\lambda_i)(4(d-3)\mathrm{tr}(g_{(4)}) + \ldots) + \frac{d-3}{d-1} l^2 L^2 \lambda_1 \, Weyl^2_{(0)} = 0.$$

There are again four cases.

The modification due to higher curvature corrections corresponds to a shift in the c coefficient of the (conformal) trace anomaly:

$$< T_\mu{}^\mu >= aE_4 - c\,Weyl^2, \qquad a \neq c.$$

Interestingly only the $Riem^2$ term in the action contributes to this shift [in agreement with Nojiri, Odintsov (1999); Blau, Gava, Narain (1999); Schwimmer, Theisen (2003)].

Scale vs. conformal invariance

- There are real-world critical phenomena about which we do not know if the corresponding fixed point is conformally or "only" scale invariant
- Do CFTs exhaust all possible second order phase transitions?

Under the assumptions of locality and unitarity:

- In 2D scale invariance implies conformal invariance ([Polchinski (1988)).
- In 4D there are strong arguments that scale invariance implies conformal invariance[Dymarsky, Komargodski, Schwimmer, Theisen (2013)]
- In higher even dimensions or in odd dimensions - answer unknown

Holography

The natural questions are:

- Can we use holography to prove "scale $=>$ conformal"?
- Can we construct examples of scale but not conformally invariant theories?
- What is the holographic dual of the virial current?

Trace anomaly as a signature of scale invariance in 4d

Scaling anomaly

In 4d scale invariant theory allows more general anomaly when coupled to the background metric:

$$T^{\mu}_{\mu} = aE_4 - c\,Weyl^2 + eR^2.$$

Presence of the R^2 term in the trace anomaly is a clear signal of a scale but not conformally invariant field theory.

Einstein-Hilbert Gravity

- R^2 anomaly in Einstein-Hilbert gravity?
- NO! Analysis by [Henningson, Skenderis (1998)] demonstrated that no R^2 anomaly may appear if the gravitational sector is described by Einstein-Hilbert gravity.

Higher derivative corrections

Generic gravitational theories with the higher derivative corrections are believed to be dual to non-unitatry field theories. There are known examples of non-unitary scale invariant theories [Riva, Cardy (2005); El-Showk, Nakayama, Rychkov (2011)]. Can one construct a holographic example of non-unitary scale invariant theory using R^2 type corrections?

Trace anomaly in 4d with higher curvature corrections

The modification due to higher curvature corrections corresponds to a shift in the c coefficient of the (conformal) trace anomaly:

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Interestingly only the $Riem^2$ term in the action contributes to this shift [see also Nojiri, Odintsov (1999); Blau, Gava, Narain (1999); Schwimmer, Theisen (2003)].

Scaling anomaly

Higher curvature correction do not realise scale invariant field theories holographically.

Chern-Simons gravity

Chern-Simons gravity in 5D is a special Lovelock type gravity ($-4\lambda_1 = -4\lambda_3 = \lambda_2 = \lambda_\star$) such that the two AdS vacua degenerate ($\lambda = 1/64$).

The coefficients

$$g_{(2)ij}, \qquad \mathrm{tr}(g_{(4)}), \qquad \nabla^j g_{(4)ij}$$

are not determined by the near-boundary analysis and cannot be associated with new degrees of freedom!

Chern-Simons gravity

What is the holographic interpretation of this phenomenon?

- There are solutions in CS gravity involving arbitrary undetermined functions [J.T. Wheeler (1986); Charmousis, J.-F. Dufaux (2002); ...]
- There are no standard perturbative expansion around general solutions
- From the Hamiltonian point of view CS gravity is a 'degenerate' and 'irregular' system. [M. Banados, L.J. Garay, M.Henneaux (1996)]

Conclusions and Open Questions

- The asymptotic structure of solutions in gravity with higher curvature corrections is very rich
- Systematic analysis of the new degrees of freedom
- In the non-degenerate cases only conformal field theories are realized - no R^2 anomaly

- Variational problem in the presence of higher curvature corrections?
- Canonical analysis of gravitational Chern-Simons in 5d