Nordic String Meeting 2016

Jacobs University Bremen

Universality in String Interactions

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based on arXiv:1602.01674 with Y-t. Huang and C. Wen

14.03.2016

Intro I: Field theories versus superstrings

open (super)string = (S)YM + ∞ massive states at $\alpha' m^2 = 1, 2, \ldots$

closed (super)string = (super)gravity + ∞ states at $\alpha' m^2 = 4, 8, \ldots$

At low energies $\alpha'(k_i \cdot k_j) \to 0$, integrate out massive states

$\Rightarrow$ effective interactions on top of (S)YM and (super)gravity

$$\mathcal{L}_{\rm eff}^{\rm open} = {\rm Tr}\big\{ F^2 + \alpha' F^3 + \alpha'^2 F^4 + \mathcal{O}(\alpha'^3) \big\}$$

$$\mathcal{L}_{\rm eff}^{\rm closed} = R + \alpha' R^2 + \alpha'^2 R^3 + \alpha'^3 R^4 + \mathcal{O}(\alpha'^4)$$

Intro I: Field theories versus superstrings

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Tensor structure & g_{string}-dependent coefficient (modulo non-pert. $\sim e^{-\frac{1}{g_{\text{string}}}}$)

$\longrightarrow$ determined by α'-expansion of scattering amplitudes.

This talk: Only tree-level interactions / genus-zero amplitudes $\sim g_{\text{string}}^{-2}$.

Intro II: Universal effective interactions

$$\mathcal{L}_{\text{eff}}^{\text{open}} = \text{Tr}\{F^2 + \alpha' F^3 + \quad \alpha'^2 F^4 + \mathcal{O}(\alpha'^3)\}$$

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In general, different string theories $\leftrightarrow$ different eff. operators, ...

Intro II: Universal effective interactions

$$\mathcal{L}_{\rm eff}^{\rm type\ I} = {\rm Tr}\{F^2 + \cancel{\alpha' F^3} + \zeta_2 \alpha'^2 F^4 + \mathcal{O}(\alpha'^3)\}$$

$$\mathcal{L}_{\rm eff}^{\rm IIA/B} = R + \cancel{\alpha' R^2} + \cancel{\alpha'^2 R^3} + \zeta_3 \alpha'^3 R^4 + \mathcal{O}(\alpha'^4)$$

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... e.g. $\alpha' F^3$ absent for type I and no $\alpha' R^2 + \alpha'^2 R^3$ for type IIA/B

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In general, different string theories $\leftrightarrow$ different eff. operators, ...

... e.g. $\alpha' F^3$ absent for type I and no $\alpha' R^2 + \alpha'^2 R^3$ for type IIA/B

This talk: Identify universal tree-level interactions at all orders in α',

KLT

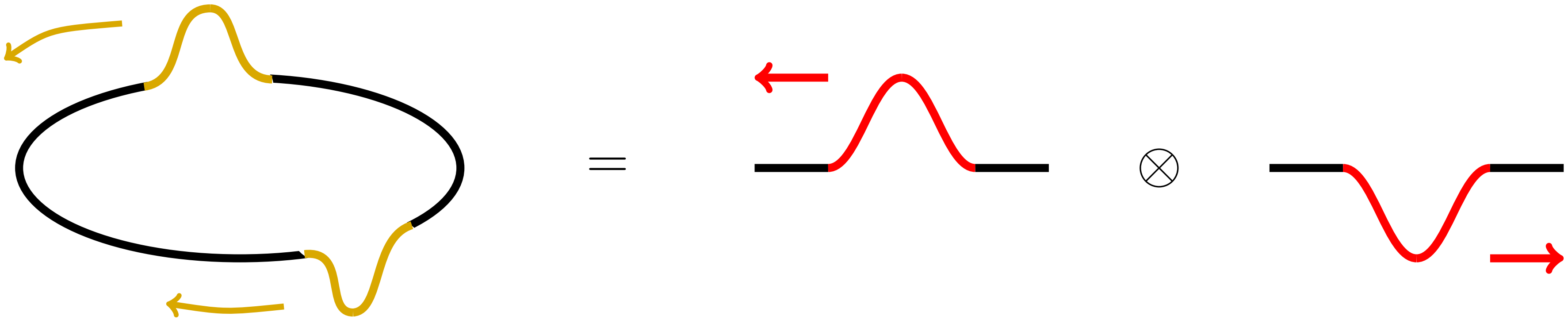
$\zeta_{\text{weight } w} \, (\alpha')^w \, \text{Tr}\{D^{2n} F^{w+2-n}\}$ common to { open superstring, open bosonic string }

$\zeta_{\text{weight } w} \, (\alpha')^w \, D^{2n} R^{w+1-n}$ common to { type II superstring, heterotic string, closed bosonic string }

Result: operators are universal **iff** $(\alpha')^w \leftrightarrow$ multiple zeta values @ weight w

[**Huang, OS, Wen 1602.01674**]

Intro III: Closed-string spectrum $\cong$ double-copy of open-string states



$\Rightarrow$ KLT-rel's: Closed-string tree $\cong$ (open-string tree) $\otimes$ (open-string tree)

[Kawai, Lewellen, Tye 1986]

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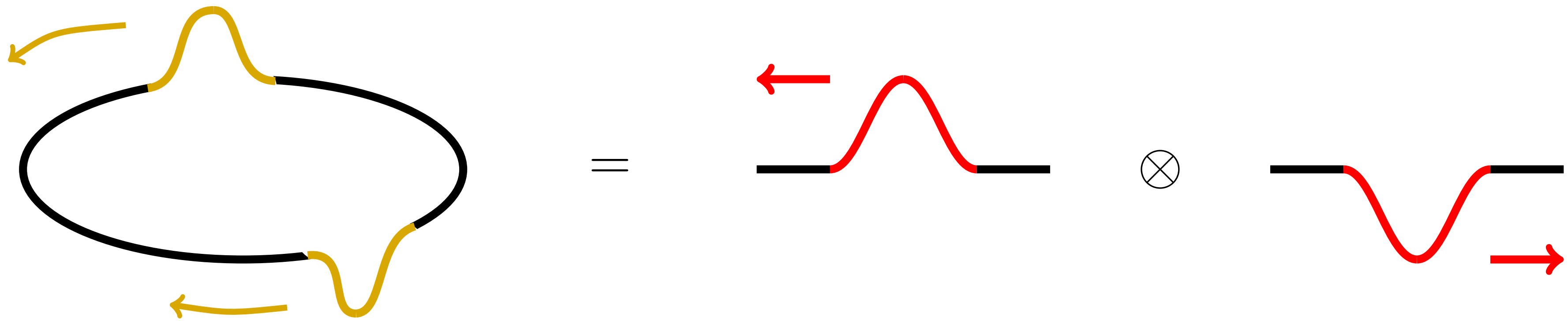

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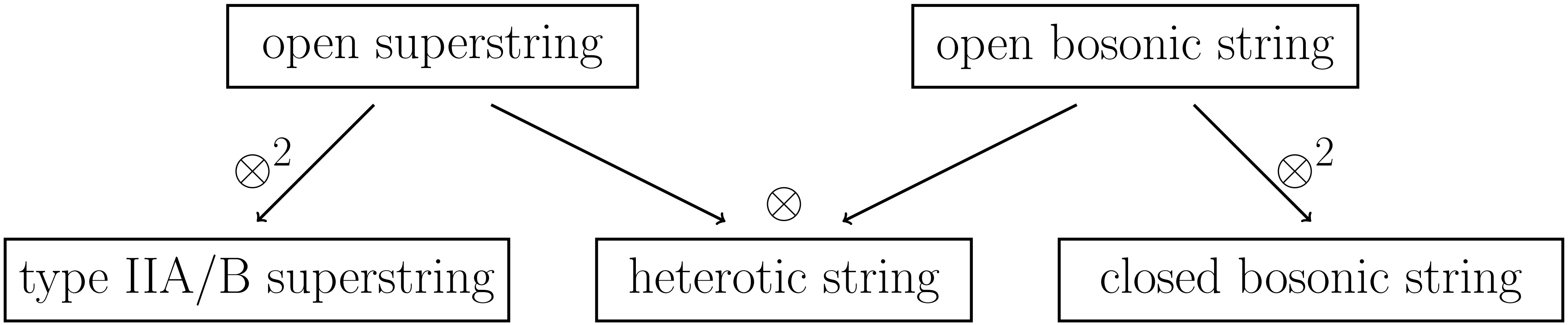
[Kawai, Lewellen, Tye 1986]

Universality of open-string tree-level amplitudes & interactions ...



... propagates to tree-level amplitudes & interactions of closed strings.

Outline

1. Four-point universality

   – open superstring versus bosonic string

   – closed superstring versus heterotic / bosonic string

2. N-point universality

   – open superstring versus bosonic string

   – closed superstring versus heterotic / bosonic string

1.1 Generalities of open-string tree amplitudes

Chan-Paton factors @ open-string endpoints → color decomposition

$$\mathcal{M}_N^{\text{open}} = \sum_{\sigma \in S_{N-1}} \text{Tr}\{\underbrace{t^1\, t^{\sigma(2)}\, t^{\sigma(3)} \ldots t^{\sigma(N)}}_{\text{gauge-group generators}}\}\, \mathcal{A}^{\text{open}}(\underbrace{1, \sigma(2), \sigma(3), \ldots, \sigma(N)}_{\text{ordering @ disk boundary}})$$

Mostly discuss color-stripped amplitude $\mathcal{A}^{\text{open}}(1, 2, \ldots, N)$:

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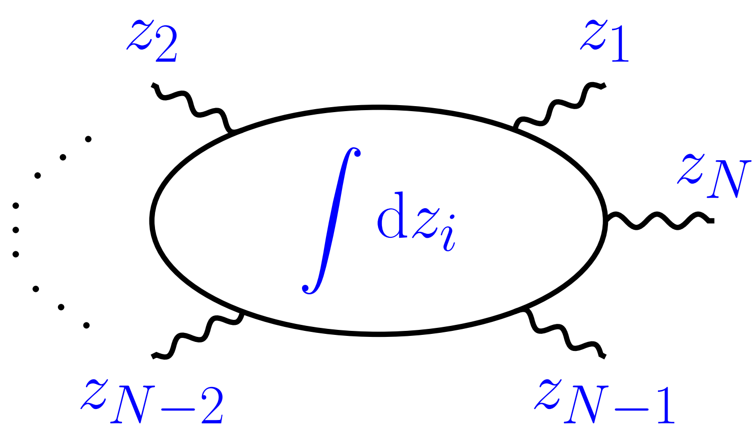
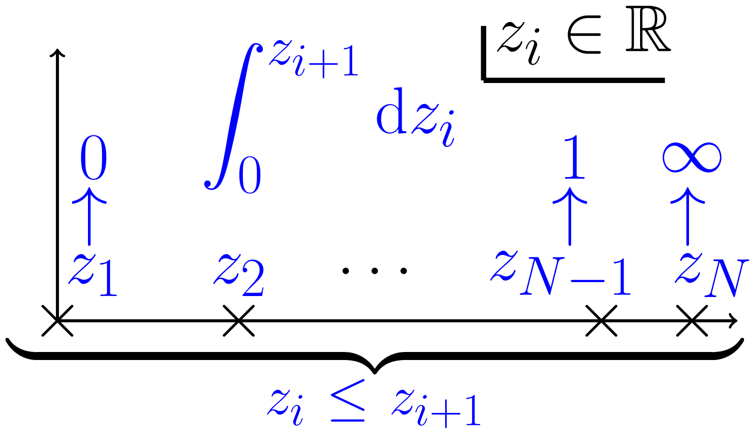


conformal symmetry →



Universal α'-corrections from iterated integrals along disk boundary,

after fixing $(z_1, z_{N-1}, z_N) \to (0, 1, \infty)$, left with $(N-3)$ integrations.

1.2 Four-point open-superstring amplitude

$$\mathcal{A}^S(1,2,3,4;\alpha') = F^{(2)}(\alpha') \times A_{\mathrm{SYM}}(1,2,3,4;\cancel{\alpha'})$$

Veneziano formfactor in terms of dim'less Mandelstams $s_{ij} \equiv \alpha'(k_i+k_j)^2$

$$\begin{aligned} F^{(2)}(\alpha') &= -s_{12}\int_0^1 \mathrm{d}x\ |x|^{s_{12}-1}|1-x|^{s_{23}} \\ &= \frac{\Gamma(1+s_{12})\Gamma(1+s_{23})}{\Gamma(1+s_{12}+s_{23})} \\ &= 1 - \underbrace{\zeta_2\, s_{12}s_{23}}_{\alpha'^2 F^4 \text{ vertex}} - \underbrace{\zeta_3\, s_{12}s_{23}s_{13}}_{\alpha'^3 D^2 F^4 \text{ vertex}} + \mathcal{O}(\alpha'^4) \end{aligned}$$

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α'-corrections accompanied by Riemann zeta values

$$\zeta_n \equiv \sum_{k=0}^{\infty} \frac{1}{k^n} \,, \qquad \text{weight } n \geq 2$$

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α'-corrections accompanied by Riemann zeta values

uniform transcendentality : $(\alpha')^n \quad \leftrightarrow \quad \zeta_n \equiv \sum_{k=0}^{\infty} \frac{1}{k^n} \, ,$ weight $n \geq 2$

Note that products $\zeta_{n_1}\zeta_{n_2}$ are said to have weight $n_1 + n_2$.

1.3 Four-point bosonic open-string amplitude

$$\mathcal{A}^B(1,2,3,4;\alpha') = F^{(2)}(\alpha') \times B(1,2,3,4;\alpha')$$

Same integral $F^{(2)}(\alpha')$ with α'-dependent kinematic factor

$$B(1,2,3,4;\alpha') \equiv A_{\text{YM}}(1,2,3,4) + (2\alpha')^2 \times s_{13}\left[\left(\frac{f_{12}f_{34}}{s_{12}^2(1-s_{12})} + \text{cyc}(2,3,4)\right) - \frac{g_1g_2g_3g_4}{s_{12}^2 s_{13}^2 s_{14}^2}\right],$$

where $\begin{cases} f_{ij} \equiv (e_i{\cdot}e_j)(k_i{\cdot}k_j) - (k_i{\cdot}e_j)(k_j{\cdot}e_i) \\ g_i \equiv (k_{i-1}{\cdot}e_i)s_{i,i+1} - (k_{i+1}{\cdot}e_i)s_{i-1,i} \end{cases}$

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After geometric-series expansion $\frac{1}{1-s_{ij}} = \sum_{n=0}^{\infty} s_{ij}^n$ with $s_{ij} = \alpha'(k_i+k_j)^2$,

corrections to $A_{\rm YM}(1,2,3,4)$ are $\mathcal{O}(\alpha')$ and have no ζ_n:

$$B(1,2,3,4;\alpha') = A_{\rm YM}(1,2,3,4) + \sum_{n=1}^{\infty}(2\alpha')^n B_n(1,2,3,4)$$

1.3 Four-point bosonic open-string amplitude

$$\mathcal{A}^B(1,2,3,4;\alpha') = \underbrace{F^{(2)}(\alpha')}_{(\alpha')^n \leftrightarrow \zeta_n} \quad \times \quad \underbrace{B(1,2,3,4;\alpha')}_{(1-s_{ij})^{-1} \Rightarrow (\alpha')^n \leftrightarrow \text{rational}}$$

non-uniform transcendentality: $(\alpha')^n \leftrightarrow \zeta_w$ @ weights $0 \leq w \leq n$.

Recall that $B(1,2,3,4;\alpha') \to A_{\rm SYM}(1,2,3,4)$ for superstring.

$\Rightarrow$ Only $(\alpha')^n \leftrightarrow \zeta_n$ common to $\mathcal{A}^S(\alpha')$ and $\mathcal{A}^B(\alpha')$

$\Rightarrow$ $\zeta_n\,(\alpha')^n\, D^{2(n-2)}\, F^4$ interactions are universal!

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Tensor structure of F^4 encoded in $A_{\rm SYM}(1,2,3,4) \ \rightarrow$ "t_8-tensor"

and contractions of covariant derivatives $D = \partial + [A, \cdot]$ obtained from

$$F^{(2)}(\alpha') = \exp\left(\sum_{n=2}^{\infty} \frac{\zeta_n}{n} (-1)^n \left[s_{12}^n + s_{23}^n - (s_{12}+s_{23})^n \right] \right)$$

1.4 Four-point type IIA/B amplitudes

KLT-rel's: Closed-string tree $\cong$ (open-string tree) $\otimes$ (open-string tree)

[Kawai, Lewellen, Tye 1986]

$$\mathcal{M}_4^S(\alpha') \sim \tilde{\mathcal{A}}^S(1,2,4,3;\alpha')\,\sin(\pi s_{12})\,\mathcal{A}^S(1,2,3,4;\alpha')$$

$$\sim \tilde{A}_{\rm YM}(1,2,4,3)\,s_{12}\,G(\alpha')\,A_{\rm YM}(1,2,3,4)$$

with Shapiro / Virasoro formfactor

$$G(\alpha') = \frac{\Gamma(1+s_{12})\Gamma(1+s_{13})\Gamma(1+s_{23})}{\Gamma(1-s_{12})\Gamma(1-s_{13})\Gamma(1-s_{23})}$$

$$= 1 - \underbrace{2\,\zeta_3\,s_{12}s_{23}s_{13}}_{\alpha'^3 R^4 \text{ vertex}} - \underbrace{\zeta_5\,s_{12}s_{23}s_{13}(s_{12}^2+s_{13}^2+s_{23}^2)}_{\alpha'^3 D^4 R^4 \text{ vertex}} + \mathcal{O}(\alpha'^6)$$

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$$G(\alpha') = \frac{\Gamma(1+s_{12})\Gamma(1+s_{13})\Gamma(1+s_{23})}{\Gamma(1-s_{12})\Gamma(1-s_{13})\Gamma(1-s_{23})}$$
$$= \exp\left(-2\sum_{k=1}^{\infty}\frac{\zeta_{2k+1}}{2k+1}\left[s_{12}^{2k+1}+s_{23}^{2k+1}+s_{13}^{2k+1}\right]\right)$$

Again – uniform transcendentality for type IIA/B: $(\alpha')^n \quad \leftrightarrow \quad \zeta_n$

1.5 Universal R^4 interactions

Superstring $\mathcal{M}^S$ versus heterotic string $\mathcal{M}^{\rm het}$ & bosonic string $\mathcal{M}^B$:

$$\left.\begin{array}{r} \mathcal{M}_4^S(\alpha') \\[2ex] \mathcal{M}_4^{\rm het}(\alpha') \\[2ex] \mathcal{M}_4^B(\alpha') \end{array}\right\} = \sin(\pi s_{12}) \times \left\{\begin{array}{l} \tilde{\mathcal{A}}^S(1,2,4,3;\alpha')\,\mathcal{A}^S(1,2,3,4;\alpha') \\[2ex] \tilde{\mathcal{A}}^B(1,2,4,3;\alpha')\,\mathcal{A}^S(1,2,3,4;\alpha') \\[2ex] \tilde{\mathcal{A}}^B(1,2,4,3;\alpha')\,\mathcal{A}^B(1,2,3,4;\alpha') \end{array}\right.$$

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Recall that $B(1,2,3,4;\alpha') = A_{\rm YM}(1,2,3,4) + \mathcal{O}(\alpha')$ without any ζ_n

$\Rightarrow$ Only $(\alpha')^n \leftrightarrow \zeta_n$ universal to closed-string trees

$\Rightarrow$ $\zeta_n\,(\alpha')^n\,D^{2(n-3)}\,R^4$ interactions are universal!

Covariant derivatives from $G(\alpha') = 1 - 2\zeta_3 s_{12} s_{23} s_{13} + \ldots$

2.1 The N-point open-superstring amplitude

$$\mathcal{A}^S(1,2,\ldots,N;\alpha') \;=\; \sum_{\sigma\in S_{N-3}} A_{\rm SYM}\big(1,\sigma(2,\ldots,N-2),N-1,N\big)\, F^\sigma(\alpha')$$

[Mafra, OS, Stieberger 1106.2645, 1106.2646]

- all polarization dependence in $(N-3)!$ field theory subamplitudes $A_{\rm SYM}$
- valid for states of $\mathcal{N}=1$ SYM in $D=10$ (all gluon and gluino helicities)
- entire α'-dependence from $(N-3)!$ iterated disk integrals

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- entire α'-dependence from $(N-3)!$ iterated disk integrals

$$F^\sigma(\alpha') = \int\limits_{0\leq z_2\leq z_3\leq\ldots\leq z_{N-2}\leq 1} \mathrm{d}z_2\,\ldots\int \mathrm{d}z_{N-2}\,\prod_{i<j}^{N-1}|z_{ij}|^{s_{ij}}\,\prod_{k=2}^{N-2}\sum_{j=1}^{k-1}\frac{s_{\sigma(j),\sigma(k)}}{z_{\sigma(j),\sigma(k)}}\Bigg|_{z_{N-1}=1}^{z_1=0}$$

- consistent with field theory limit: $F^\sigma(\alpha'\to 0) \;=\; \delta^\sigma_{(2,3,\ldots,N-2)}+\ldots$

2.2 Multiple zeta values (MZVs)

α'-dependence of $\mathcal{A}^S(\alpha') = F(\alpha') \cdot A_{\rm SYM}$ involves MZVs $(n_r \geq 2)$

$$\zeta_n \ \equiv \ \sum_{k=1}^{\infty} k^{-n} \quad \longrightarrow \quad \zeta_{n_1,n_2,\ldots,n_r} \ \equiv \sum_{0<k_1<\ldots<k_r} k_1^{-n_1} \, k_2^{-n_2} \, \ldots \, k_r^{-n_r}$$

conjecturally graded by weight $w = \sum_{j=1}^{r} n_j$.

Uniform transcendentality of N-point superstring amplitude:

$$(\alpha')^w \text{ order of } F^\sigma(\alpha') \quad \leftrightarrow \quad \text{MZVs of weight } w$$

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Uniform transcendentality of N-point superstring amplitude:

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Inductive proof via Drinfeld associator $\Phi(\alpha')$ of uniform transcendentality:

$$F_{N\text{-pt}}(\alpha') = \Phi(\alpha') \cdot F_{(N-1)\text{-pt}}(\alpha')$$

[Brödel, OS, Stieberger, Terasoma 1304.7304]

2.3 The N-point bosonic open-string amplitude

Same integral basis F^σ for superstring $\mathcal{A}^S(\alpha')$ and bosonic string $\mathcal{A}^B(\alpha')$:

$$\left.\begin{array}{l} \mathcal{A}^S(1,2,\ldots,N;\alpha') \\ \\ \mathcal{A}^B(1,2,\ldots,N;\alpha') \end{array}\right\} = \sum_{\sigma\in S_{N-3}} F^\sigma(\alpha') \left\{\begin{array}{l} A_{\rm SYM}\big(1,\sigma(2,\ldots,N-2),N-1,N\big) \\ \\ B\big(1,\sigma(2,\ldots,N-2),N-1,N;\alpha'\big) \end{array}\right.$$

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[Huang, OS, Wen 1602.01674]

α'-dependence of $B(\ldots;\alpha')$: geometric series $(1-s_{ij})^{-1} = \sum_{k=0}^\infty s_{ij}^k$

$$B(1,2,\ldots,N;\alpha') = A_{\rm YM}(1,2,\ldots,N) + \sum_{k=1}^{\infty} (2\alpha')^k B_k(1,2,\ldots,N)$$

only rational numbers, no MZVs!

$\Rightarrow$ Only $(\alpha')^w \leftrightarrow \text{MZV}\big|_{\text{weight } w}$ common to $\mathcal{A}^S(\alpha')$, $\mathcal{A}^B(\alpha')$

$\Rightarrow$ $\text{MZV}_w\,(\alpha')^w\,D^{2m}\,F^{w+2-m}$ interactions are universal!

2.4 The N-point closed superstring amplitude

Insert $\mathcal{A}^S(\alpha') = F(\alpha') \cdot A_{\rm SYM}$ into KLT-relations

$$\mathcal{M}^S(\alpha') = \sum_{\substack{\sigma,\rho,\tau \\ \in S_{N-3}}} \tilde{A}_{\rm SYM}(1,\sigma,N,N{-}1)\,(S_0)_\sigma{}^\rho\, G_\rho{}^\tau(\alpha')\, A_{\rm SYM}(1,\tau,N{-}1,N)$$

$S_0 \equiv (N-3)! \times (N-3)!$ matrix with entries of order $(k_i \cdot k_j)^{N-3}$

known from supergravity amplitude where $G_\rho{}^\tau(\alpha') \to \delta^\tau_\rho$, e.g.

$$(S_0)_\sigma{}^\rho \big|_{\rm 4pt} = s_{12} \ , \qquad (S_0)_\sigma{}^\rho \big|_{\rm 5pt} = \begin{pmatrix} s_{12}(s_{13}+s_{23}) & s_{12}s_{13} \\ s_{12}s_{13} & s_{13}(s_{12}+s_{23}) \end{pmatrix}$$

All α' dependence in $(N-3)! \times (N-3)!$ matrix $G(\alpha')$,

entries $1+\mathcal{O}(\alpha'^3)$ have uniform transcendentality.

2.4 The N-point closed superstring amplitude

Promote $F^\sigma(\alpha')$ to an $(N-3)! \times (N-3)!$ matrix via

$$\mathcal{A}^S(1,\rho,N-1,N;\alpha') = \sum_{\sigma\in S_{N-3}} F_\rho{}^\sigma(\alpha') A_{\rm SYM}(1,\sigma,N-1,N)$$

α'-expansion of $F_\rho{}^\sigma(\alpha')$ has matrix-multiplicative structure,

$$\begin{aligned} F(\alpha') &= \mathbb{1} + \zeta_2 P_2 + \zeta_3 M_3 + \zeta_4 P_4 + \zeta_5 M_5 + \underline{\zeta_2 P_2 \zeta_3 M_3} \\ &+ \zeta_6 P_6 + \underline{\tfrac{1}{2}\zeta_3^2 M_3^2} + \zeta_7 M_7 + \underline{\zeta_2 P_2 \zeta_5 M_5} + \underline{\zeta_4 P_4 \zeta_3 M_3} + \mathcal{O}(\alpha'^8) \end{aligned}$$

[OS, Stieberger 1205.1516]

entries of $(N-3)! \times (N-3)!$ matrices P_w, M_w are

degree-w polynomials in s_{ij} with rational coefficients!

Closed-string α'-exp. $\mathcal{M}^S(\alpha') = \tilde{A}_{\rm SYM} \cdot S_0 \cdot G(\alpha') \cdot A_{\rm SYM}$ is "filtered":

$$G(\alpha') = 1 + 2\,\zeta_3 M_3 + 2\,\zeta_5 M_5 + 2\,\zeta_3^2 M_3^2 + 2\,\zeta_7 M_7 + \ldots$$

2.4 The N-point closed superstring amplitude

Curvature interactions $D^{2m}R^n$ determined by

- bilinears of SYM trees in $\mathcal{M}^S(\alpha') = \tilde{A}_{\rm SYM} \cdot S_0 \cdot G(\alpha') \cdot A_{\rm SYM}$
- open-string matrices M_w of order s_{ij}^w, see `http://mzv.mpp.mpg.de`

$$G(\alpha') = 1 + \underbrace{2\,\zeta_3\, M_3}_{\alpha'^3 R^4} + \overbrace{2\,\zeta_5\, M_5}^{\alpha'^5(D^4R^4 \oplus D^2R^5 \oplus R^6)} + \ldots + \underbrace{\tfrac{2}{5}\,\zeta_{3,3,5}\,\big[M_3, [M_5, M_3]\big]}_{\alpha'^{11}(D^{14}R^5 \oplus D^{12}R^6 \oplus \ldots \oplus R^{12})} + \ldots$$

[OS, Stieberger 1205.1516]

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[OS, Stieberger 1205.1516]

Selection rules on MZVs in $F(\alpha') \to G(\alpha')$ is single-valued (sv) projection

$$\text{sv}: \zeta_{2n},\ \zeta_{2n+1},\ \zeta_{3,5},\ \zeta_{3,7},\ \zeta_{3,3,5},\ \ldots \longrightarrow \cancel{\zeta_{2n}},\ \zeta_{2n+1},\ \cancel{\zeta_{3,5}},\ \cancel{\zeta_{3,7}},\ \zeta_{3,3,5},\ \ldots$$

[Stieberger 1310.3259]

2.5 Universal closed-string interactions

Same sphere integrals $G(\alpha')$ for all closed-string trees

$$\text{type II:}\quad \mathcal{M}^S(\alpha') = \tilde{A}_{\text{SYM}}\, S_0\, G(\alpha')\, A_{\text{SYM}}$$

$$\text{heterotic:}\quad \mathcal{M}^{\text{het}}(\alpha') = \tilde{A}_{\text{SYM}}\, S_0\, G(\alpha')\, B(\alpha')$$

$$\text{bosonic:}\quad \mathcal{M}^B(\alpha') = \tilde{B}(\alpha')\, S_0\, G(\alpha')\, B(\alpha')\ .$$

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Since $B(\alpha') = A_{\rm YM} + \mathcal{O}(\alpha')$ has rational coefficients,

$\Rightarrow$ Only $(\alpha')^w \leftrightarrow \text{MZV}\big|_{\text{weight } w}$ universal to closed-string trees

$\Rightarrow$ $\text{(sv-)MZV}_w\, (\alpha')^w\, D^{2m}\, R^{w+1-m}$ interactions are universal!

[Huang, OS, Wen 1602.01674]

Simplest non-universal interaction: $\alpha' R^2$ of heterotic and bosonic string.

Summary

$(\alpha')^w \, \mathrm{Tr}\{D^{2m} \, F^{w+2-m}\}$ interactions are common to

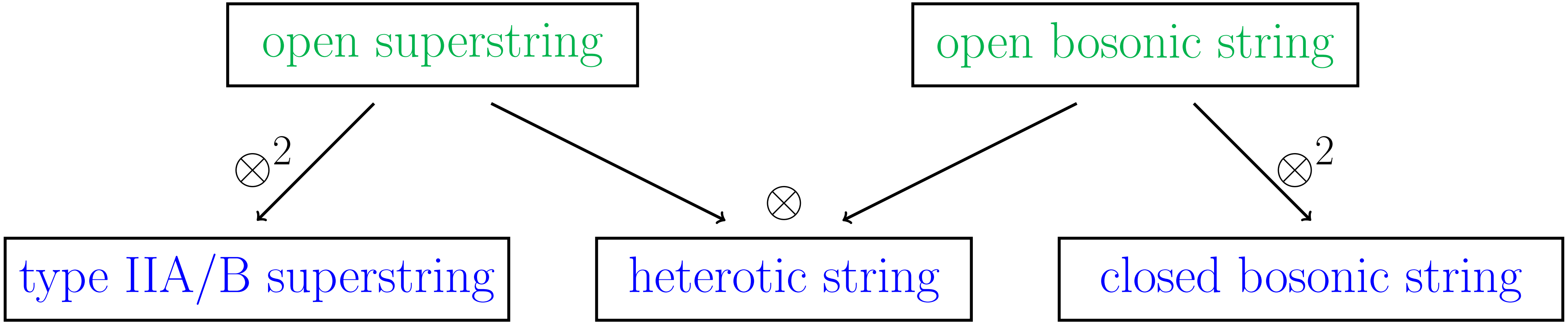
open superstring	open bosonic string

... iff their coefficients obey uniform transcendentality

$$(\alpha')^w \quad \leftrightarrow \quad \text{MZVs} \;\; \zeta_{n_1,n_2,\ldots,n_r} \text{ @ weight } w = n_1 + n_2 + \ldots + n_r$$

Summary

$(\alpha')^w \, \mathrm{Tr}\{D^{2m} F^{w+2-m}\}$ & $(\alpha')^w \, D^{2m} R^{w+1-m}$ interactions are universal



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$$(\alpha')^w \quad \leftrightarrow \quad \text{MZVs } \ \zeta_{n_1,n_2,\ldots,n_r} \ \text{@ weight } w = n_1 + n_2 + \ldots + n_r$$

Thank you for your attention !