One-point functions in AdS/dCFT

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Joint work with I. Buhl Mortensen, C. Kristjansen and K. Zarembo, arXiv:1506.06958 and 1512.02532

Outline

Introduction

AdS/CFT

String theory on $\mathrm{AdS}_5 \times \mathrm{S}^5 \Leftrightarrow \mathcal{N} = 4$ super Yang–Mills

Link with integrable spin chains [Minahan Zarembo 02]

- Dilatation operator $\Leftrightarrow$ integrable spin chain Hamiltonian
- Progress in spectral problem *i.e.* two-point functions

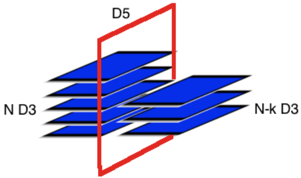
Open problems (to some degree)

- three-point functions
- non-planar
- one-point functions in certain brane set-ups (dCFT)

Use integrability to compute one-point functions in a dCFT

String theory picture

D3-D5 probe brane set-up.



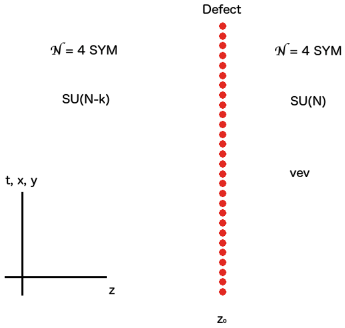
- $D5$ brane $\sim \mathrm{AdS}_4 \times \mathrm{S}^2$
- $D3$ brane $\sim \mathrm{M}_4$

The D3 and D5 overlap in M_3 which is the defect

Defect CFT

[deWolfe,Freedman,Ooguri 01]

Defect $\to$ 3D fields



Tree-level $\Phi^{cl}_{4,5,6} = \Psi_A = A_\mu = 0$ and z is distance to defect

EOM $\frac{d^2\Phi^{cl}_i}{dz^2} = [\Phi^{cl}_j, [\Phi^{cl}_j, \Phi^{cl}_i]]$ [Constable,Myers,Tafjord 99]

Classical solution

Solution via k-dim $SU(2)$ representation t_i

$$\Phi_i^{\mathrm{cl}} = \frac{1}{z}\begin{pmatrix} (t_i)_{k\times k} & 0_{k\times(N-k)} \\ 0_{(N-k)\times k} & 0_{(N-k)\times(N-k)} \end{pmatrix}$$

Non-trivial vev $\Rightarrow$ scalar operators

$$\mathcal{O} = \Psi^{i_1\cdots}\,\mathrm{tr}\,\Phi_{i_1}\ldots \Rightarrow \langle\mathcal{O}\rangle = \Psi^{i_1\ldots i_L}\,\frac{\mathrm{tr}\, t_{i_1}\ldots t_{i_L}}{z^L} \equiv \frac{\langle \mathrm{MPS}|\mathcal{O}\rangle}{z^L}$$

One-point function can be written as overlap

- $|\mathrm{MPS}\rangle$ is state associated with the defect.
- Calculate $C_k \equiv \frac{\langle \mathrm{MPS}|\mathcal{O}\rangle}{\sqrt{\langle\mathcal{O}|\mathcal{O}\rangle}}$

Overlap

Restrict to $SU(2)$ subsector

$$Z = \Phi_1 + i\Phi_4 \sim \uparrow, \qquad W = \Phi_2 + i\Phi_5 \sim \downarrow .$$

At planar level and at one-loop conformal operators in $SU(2) \Leftrightarrow$ eigenstates of XXX spin chain (Bethe states) [Minahan,Zarembo 02]

$$\mathcal{O} \sim \mathrm{tr}\, ZWWZWZWW \ldots \sim |\uparrow\downarrow\downarrow \ldots\rangle$$

Defect state (Matrix Product State) in $SU(2)$ sector

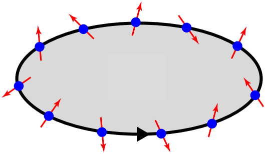
$$\langle \mathrm{MPS}| = \mathrm{tr} \prod_{\ell=1}^{L} \left[\langle \uparrow |_\ell \otimes t_1 + \langle \downarrow |_\ell \otimes t_2 \right]$$

Find general framework to compute overlap

XXX spin chain

XXX spin chain

Heisenberg spin chain (model for magnetism):



Consider L spin $\frac{1}{2}-$particles

Heisenberg spin chain

- Hilbert space $V = \bigotimes_{i=1}^{L} V_i$ where $V_i = \mathbb{C}^2$
- Hamiltonian $\mathcal{H} = \sum_{i=1}^{L} S_i \cdot S_{i+1}$

Periodic boundary conditions $L + i \equiv i$

Bethe Ansatz

Eigenstates of $\mathcal{H}$ follow from Bethe Ansatz [Bethe 31]

Bethe Ansatz

- Vacuum state $|0\rangle = |\uparrow\uparrow\ldots\rangle$
- Plane-wave type excitations with rapidity $u = \frac{1}{2}\cot\frac{p}{2}$

$$|u\rangle = \sum_{n=1}^{L} e^{ipn}\,|\uparrow\ldots\downarrow_n\ldots\rangle$$

- General M spins flipped $\rightarrow$ scattering phase θ

$$|\{u_i\}\rangle = N \sum_{\sigma\in S_M}\sum_{n_i} e^{i\sum_m p_{\sigma_m} n_m} S_\sigma |\ldots\downarrow_{n_1}\ldots\downarrow_{n_2}\ldots\rangle,$$

periodic BC $\Rightarrow$ Bethe equations

$$e^{ip_n L} = \prod_{j\neq n} S_{nj}$$

Problem

States from dCFT in correspondence with states $|\{u_i\}\rangle$

Then

- $\langle \mathrm{MPS}| = \mathrm{tr} \prod_{\ell=1}^{L} [\langle \uparrow |_\ell \otimes t_1 + \langle \downarrow |_\ell \otimes t_2]$

Formulation in XXX

$$C_k(\{u_i\}) \equiv \frac{\langle \mathrm{MPS}|\{u_i\}\rangle}{\sqrt{\langle \{u_i\}|\{u_i\}\rangle}}$$

u_i solutions Bethe equations

$\langle \{u_i\}|\{u_i\}\rangle$ known [Gaudin 76]

Overlap

General

General considerations

- C vanishes if L or M is odd
  $\Rightarrow$ easy to see for $k=2$ since $\{t_i, t_j\} = 0$ and $t_i^2 \sim 1$
- C is zero if $P = \sum p_i \neq 0$
  $\Rightarrow \langle \mathrm{MPS}|\{u_i\}\rangle = (\langle \mathrm{MPS}|e^{iP})|\{u_i\}\rangle = \langle \mathrm{MPS}|(e^{iP}|\{u_i\}\rangle) = e^{iP}\langle \mathrm{MPS}|\{u_i\}\rangle$
- C is only non-zero if $\{u_i\} = \{-u_i\}$
  $\Rightarrow$ the odd conserved charges vanish on the defect state

Restrict to states $|\{u_1, -u_1, \ldots, u_{\frac{M}{2}}, -u_{\frac{M}{2}}\}\rangle$

Determinant

For $k = 2$: compact expression in $\frac{M}{2} \times \frac{M}{2}$ matrices

$$K^{\pm}_{jk} = \frac{2}{1 + (u_j - u_k)^2} \pm \frac{2}{1 + (u_j + u_k)^2} \,,$$

and

$$G^{\pm}_{jk} = \left(\frac{L}{u_j^2 + \frac{1}{4}} - \sum_n K^{+}_{jn} \right) \delta_{jk} + K^{\pm}_{jk} .$$

$$C_2(\{u_j\}) = 2^{1-L} \left(\prod_j \frac{u_j^2 + \frac{1}{4}}{u_j^2} \, \frac{\det G^{+}}{\det G^{-}} \right)^{\frac{1}{2}} .$$

Proof: Equivalent to (raised) Néel state $|\uparrow\downarrow\ldots\rangle + |\downarrow\uparrow\ldots\rangle$

$$\langle \mathrm{MPS} | \mathcal{O} \rangle \simeq \langle \mathrm{Neel} | \mathcal{O} \rangle$$

Formula from cond-mat literature e.g. [Posgay] [Brockman et al. 14]

Determinant

For general k result factorizes

$$\frac{C_k(\{u_j\})}{C_2(\{u_j\})} = 2^{L-1} \sum_{j=\frac{1-k}{2}}^{\frac{k-1}{2}} j^L \prod_{i=1}^{\frac{M}{2}} \frac{u_i^2(u_i^2+\frac{k^2}{4})}{\left[u_i^2+(j-\frac{1}{2})^2\right]\left[u_i^2+(j+\frac{1}{2})^2\right]}$$

Follows from recursion relation with transfer matrix T

$$C_{k+2} = T(\tfrac{ik}{2})C_k - \left(\frac{k+1}{k-1}\right)^L C_{k-2}.$$

Proof Computation via Lax matrices

Agreement with string theory computation $M = 0$ and $L, k, \lambda \to \infty$

Conclusions

Conclusions

Conclusions

- Closed formula for one point functions in the D3-D5 dCFT
- Write it as overlap with a 'defect state'
- Determinant formula

Open questions

- Comparison with string theory
- Other branes set-ups?
- Loops and other sectors

Thank you